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ELECTROMAGNETIC FIELDS IN AN N-LAYER ANISOTROPIC HALF-SPACE†

DOUGLAS P. O'BRIEN* AND H. F. MORRISON**

From Maxwell's equations and Ohm's law for a horizontally anisotropic medium, it may be shown that two independent plane wave modes propagate perpendicular to the plane of the anisotropy. Boundary conditions at the interfaces in an n-layered model permit the calculation, through successive matrix multiplications, of the fields at the surface in terms of the fields propagated into the basal infinite half space. Specifying the magnetic field at the surface allows the calculation of the resultant electric fields, and the calculation of the entries of a tensor impedance relationship. These calculations have been programmed for the digital computer and an interpretation of impedances obtained from field measurements may thus be made in terms of the anisotropic layering. In addition, apparent resistivities in orthogonal directions have been calculated for specific models and compared to experimental data. It is apparent that the large scatter of observed resistivities can be caused by small changes in the polarization of the magnetic field.

INTRODUCTION

Magnetotelluric studies have, in general, shown that electrical conductivities within the earth are strongly directional. This is particularly evident in the apparent resistivity measurements in orthogonal directions carried out by Fournier (1962, 1963a, 1963b), Bostick and Smith (1962), and Hopkins and Smith (1966), to mention only a few. For a uniform plane wave normally incident on an anisotropic half space, Cantwell (1960) derived expressions for the E and H fields in terms of an admittance tensor:

$$\begin{vmatrix} H_x \\ H_y \end{vmatrix} = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix} \cdot \begin{vmatrix} E_x \\ E_y \end{vmatrix}.$$

Later, Mann (1965) discussed the theory of a plane wave incident on an anisotropic half space. As Cantwell pointed out in his study, the advantage of the admittance (or impedance) tensor notation is that the tensor entries uniquely characterize the half space, regardless of the model chosen. Bostick and Smith (1962) obtained apparent resistivities from the values of Y_{12} and Y_{21} when the measuring axes had been computationally rotated to a position which minimized the diagonal entries. If it were assumed that the earth model could be described by an anisotropic half space, these apparent resistivities would be the principle values of this anisotropy. They could equally well represent the effective resistivities parallel and perpendicular to a linear inhomogeneity.

In addition, it is apparent from the field results that the "anisotropy" is frequency dependent indicating changes of anisotropy with depth.

In this study it is assumed that the directional properties of resistivity within the earth can be described by a simple anisotropy and, further, that changes in anisotropy with depth can be approximated by an n-layer model. While this may be a rather idealistic approach to actual geologic situation, it is certainly a good approximation for studying the gross features of magnetotelluric field results. In the first part of the study we develop the relationships between E and H, in terms of tensor impedances, at the surface of an n-layered medium where any or all of the layers may be anisotropic. These results then allow specific model calculations for the interpretation of experimentally determined impedance tensor entries. In addition, apparent resistivities can be calculated in two orthogonal directions; and it is shown that the very large scatter in these values obtained in conventional magnetotelluric soundings may be accounted for by small changes in the azimuth of polarization

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* University of California, Berkeley; presently with Varian Associates, Palo Alto, California.

** Department of Mineral Technology, University of California, Berkeley.

of the magnetic field at the surface of such a model.

THEORY

The model consists of n homogeneous anisotropic plane parallel layers underlain by a homogeneous isotropic half space. In all layers, one of the principle axes of the conductivity anisotropy is perpendicular to the interfaces, and this is defined as the z axis. The other two lie in the x–y plane, parallel to the interfaces. The principle horizontal conductivity σ_1 lies at an angle ϕ to the x axis (Figure 1).

We shall investigate a uniform plane electromagnetic wave propagating along the z axis. The conductivity along the z axis will thus not enter our calculations since, by the definition of such a wave, $J_z=0$. We may then write Ohm's law relative to a fixed $x-y$ coordinate frame

$$|\mathbf{J}| = |\sigma_{l,m}|\,|\mathbf{E}|$$

where

$$|\sigma_{l,m}| = \begin{vmatrix} \sigma_1 \cos^2\phi + \sigma_2 \sin^2\phi & (\sigma_2 - \sigma_1)\sin\phi\cos\phi \\ (\sigma_2 - \sigma_1)\sin\phi\cos\phi & \sigma_1 \sin^2\phi + \sigma_2\cos^2\phi \end{vmatrix}. \tag{1}$$

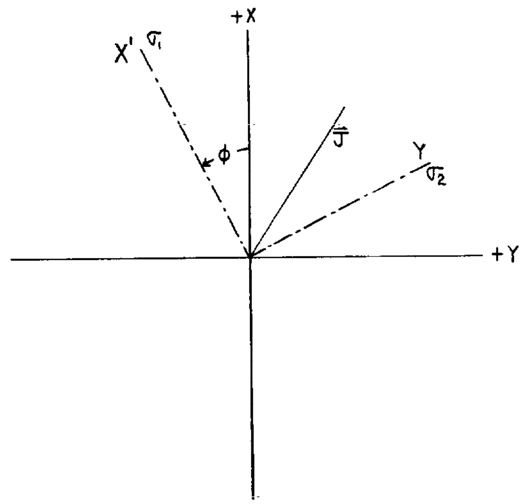

FIG. 1. Relationship of conductivity anisotropy to coordinate axes.

Since we have assumed plane waves propagating along the z axis, we may assume a solution of the form

$$\left.\begin{matrix}\mathbf{E}\\ \mathbf{H}\end{matrix}\right\} = \left.\begin{matrix}\mathbf{E}\\ \mathbf{H}\end{matrix}\right\} e^{-j\omega t + jkz} \tag{2}$$

for the resulting fields.

If we further restrict our discussion to typical earth materials at low frequencies, displacement currents are negligible in comparison with the conduction currents. From Maxwell's equations,

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \times \mathbf{H} = \sigma_{l,m}\mathbf{E}$$

we may write for the tangential components, using the previous form for solution (2)

$$\begin{aligned} jkE_y &= -j\omega\mu_0 H_x \\ jkE_x &= j\omega\mu_0 H_y \\ -jkH_y &= \sigma_{11}E_x + \sigma_{12}E_y \\ jkH_x &= \sigma_{21}E_x + \sigma_{22}E_y \end{aligned} \tag{3}$$

where the σ's are determined by the entires of (1). For a solution to the set of equations (3) to exist, the determinant of the coefficients of the field quantities E_x, E_y, etc., must be set equal to zero. This allows a solution for k,

$$k^2 = \frac{j\omega\mu_0}{2}\left\{(\sigma_{11}+\sigma_{22}) \pm \left[(\sigma_{11}+\sigma_{22})^2 + 4(\sigma_{12}\sigma_{21} - \sigma_{11}\sigma_{22})\right]^{1/2}\right\}$$

and upon substitution of the $\sigma_{l,m}$'s from (1) we find that

$$\begin{aligned} k_1 &= \pm\,(\sigma_1\omega\mu_0)^{1/2}e^{j(\pi/4)}, \quad \text{and} \\ k_2 &= \pm\,(\sigma_2\omega\mu_0)^{1/2}e^{j(\pi/4)}. \end{aligned} \tag{4}$$

Each of the values for k corresponds to a linearly independent solution for the tangential fields. The plus or minus sign refers to waves propagating in the plus z and minus z directions, respectively. The subscripts 1 and 2 indicate that two independent modes can propagate in either direction along z. The propagation constants for each mode are identical to those derived for an infinite

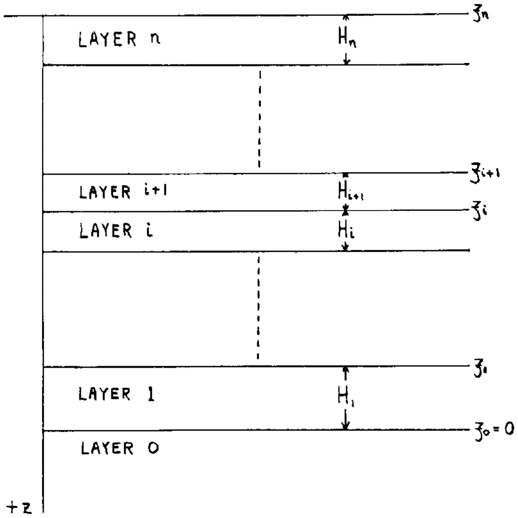

FIG. 2. n layer model.

homogeneous isotropic medium with conductivity σ_1 or σ_2.

Let us now specialize this development to a layered sequence of anisotropic conductors as shown in Figure 2. For a horizontal field component propagating in layer i of Figure 2, let the propagation constant be given by

$$\pm \overset{i}{k}_{m=1,2}$$

the subscript m referring to the mode corresponding to either σ_1 or σ_2. The following notation will be used for the field quantities. For a field quantity measured along either the x or y axis in the ith layer, whose propagation constant is $\overset{i}{k}_n$,the amplitude is represented by

$$\pm\overset{i}{E}_{x_m}, \quad \pm\overset{i}{E}_{y_m}, \quad \pm\overset{i}{H}_{x_m}, \quad \text{or} \quad \pm\overset{i}{H}_{y_m}$$

where the plus or minus superscript indicates propagation in the plus or minus z direction respectively. For example, $^-\overset{i}{E}_{x_1}$ represents the x component of the electric field propagating in the negative z direction in mode 1 and in the ith layer.

Boundary conditions between each layer are determined by the identities:

$$\mathbf{n} \times (\overset{i}{\mathbf{E}}_{\text{TOTAL}} - \overset{i+1}{\mathbf{E}}_{\text{TOTAL}}) = 0$$

$$\mathbf{n} \times (\overset{i}{\mathbf{H}}_{\text{TOTAL}} - \overset{i+1}{\mathbf{H}}_{\text{TOTAL}}) = 0 \qquad (5)$$

where $\mathbf{n}$ is the unit vector normal to the interface.

It is important to note that the vectors $\mathbf{E}_{\text{TOTAL}}$ and $\mathbf{H}_{\text{TOTAL}}$ are the superposition of the downward and upward propagating solutions, and each of these solutions is, in turn, the superposition of the two independent conductivity modes discussed previously.

By definition, the electric components of a plane wave propagating along z in the ith layer are given by:

$$\begin{aligned}
{}^+E_{x_1}{}^i &= {}^+E_{x_1}{}^i e^{jk_1^i z} & {}^+E_{x_2}{}^i &= {}^+E_{x_2}{}^i e^{jk_2^i z} \\
{}^-E_{x_1}{}^i &= {}^-E_{x_1}{}^i e^{-jk_1^i z} & {}^-E_{x_2}{}^i &= {}^-E_{x_2}{}^i e^{-jk_2^i z} \\
{}^+E_{y_1}{}^i &= {}^+E_{y_1}{}^i e^{jk_1^i z} & {}^+E_{y_2}{}^i &= {}^+E_{y_2}{}^i e^{jk_2^i z} \\
{}^-E_{y_1}{}^i &= {}^-E_{y_1}{}^i e^{-jk_1^i z} & {}^-E_{y_2}{}^i &= {}^+E_{y_2}{}^i e^{-jk_2^i z}
\end{aligned} \qquad (6)$$

from (3), the corresponding magnetic components are represented by:

$$\begin{aligned}
{}^+H_{x_1}{}^i &= \frac{-k_1{}^i}{\omega\mu_0}\,{}^+E_{y_1}{}^i & {}^+H_{x_2}{}^i &= \frac{-k_2{}^i}{\omega\mu_0}\,{}^+E_{y_2}{}^i \\
{}^-H_{x_1}{}^i &= \frac{k_1{}^i}{\omega\mu_0}\,{}^-E_{y_1}{}^i & {}^-H_{x_2}{}^i &= \frac{k_2{}^i}{\omega\mu_0}\,{}^-E_{y_2}{}^i \\
{}^+H_{y_1}{}^i &= \frac{k_1{}^i}{\omega\mu_0}\,{}^+E_{x_1}{}^i & {}^+H_{y_2}{}^i &= \frac{k_2{}^i}{\omega\mu_0}\,{}^+E_{x_2}{}^i \\
{}^-H_{y_1}{}^i &= \frac{-k_1{}^i}{\omega\mu_0}\,{}^-E_{x_1}{}^i & {}^-H_{y_2}{}^i &= \frac{-k_2{}^i}{\omega\mu_0}\,{}^-E_{x_2}{}^i
\end{aligned} \qquad (7)$$

where the factor exp $(-j\omega t)$ has been omitted for simplicity in (6) and (7).

From (3), (6), and (7), the relation between the individual electric components in the ith medium is given by:

$$
\begin{aligned}
{}^{+}E_{y_1}{}^{i} &= -\tan\phi_i{}^{+}E_{x_1}{}^{i}\\
{}^{+}E_{x_2}{}^{i} &= \tan\phi_i{}^{+}E_{y_2}{}^{i}\\
{}^{-}E_{y_1}{}^{i} &= -\tan\phi_i{}^{-}E_{x_1}{}^{i}\\
{}^{-}E_{x_2}{}^{i} &= \tan\phi_i{}^{-}E_{y_2}{}^{i}
\end{aligned}
\tag{8}
$$

where ϕ_i is the direction of the major conductivity lineation relative to the fixed x axis at the surface.

Superposing equations (6) (7) and (8), we have for the electric and magnetic components of a wave propagated in both the negative and positive z direction:

Thus, the four field components in an anisotropic medium can be expressed in terms of the four independent variables ${}^{+}E_{x_1}{}^{i}$, ${}^{-}E_{x_1}{}^{i}$, ${}^{-}E_{y_2}{}^{i}$, and ${}^{-}E_{y_2}{}^{i}$.

The $\exp(-j\omega t)$ time dependence is implied in equation (11).

Let z_i be the interface between the i and $i+1$ layers (Figure 2). Boundary conditions require that the total horizontal electric and magnetic components be continuous at z_i, i.e.

$$
\begin{aligned}
H_{x\mathrm{TOTAL}}^{i+1} &= H_{x\mathrm{TOTAL}}^{i}\\
H_{y\mathrm{TOTAL}}^{i+1} &= H_{y\mathrm{TOTAL}}^{i}\\
E_{x\mathrm{TOTAL}}^{i+1} &= E_{x\mathrm{TOTAL}}^{i}\\
E_{y\mathrm{TOTAL}}^{i+1} &= E_{y\mathrm{TOTAL}}^{i}
\end{aligned}
$$

PROPAGATION VERTICALLY DOWNWARDS

$$
\begin{aligned}
{}^{+}E_{y}{}^{i} &= {}^{+}E_{y_2}{}^{i}e^{jk_2{}^{i}z} - \tan\phi_i{}^{+}E_{x_1}{}^{i}e^{jk_1{}^{i}z}\\
{}^{+}E_{x}{}^{i} &= {}^{+}E_{x_1}{}^{i}e^{jk_1{}^{i}z} + \tan\phi_i{}^{+}E_{y_2}{}^{i}e^{jk_2{}^{i}z}\\
{}^{+}H_{x}{}^{i} &= \frac{k_1{}^{i}}{\omega\mu_0}\tan\phi_i{}^{+}E_{x_1}{}^{i}e^{jk_1{}^{i}z} - \frac{k_2{}^{i}}{\omega\mu_0}{}^{+}E_{y_2}{}^{i}e^{jk_2{}^{i}z}\\
{}^{+}H_{y}{}^{i} &= \frac{k_1{}^{i}}{\omega\mu_0}{}^{+}E_{x_1}{}^{i}e^{jk_1{}^{i}z} + \frac{k_2{}^{i}}{\omega\mu_0}\tan\phi_i{}^{+}E_{y_2}{}^{i}e^{jk_2{}^{i}z}.
\end{aligned}
\tag{9}
$$

PROPAGATION VERTICALLY UPWARDS

$$
\begin{aligned}
{}^{-}E_{y}{}^{i} &= {}^{-}E_{y_2}{}^{i}e^{-jk_2{}^{i}z} - \tan\phi_i{}^{-}E_{x_1}{}^{i}e^{-jk_1{}^{i}z}\\
{}^{-}E_{x}{}^{i} &= {}^{-}E_{x_1}{}^{i}e^{-jk_1{}^{i}z} + \tan\phi_i{}^{-}E_{y_2}{}^{i}e^{-jk_1{}^{i}z}\\
{}^{-}H_{x}{}^{i} &= -\frac{k_1{}^{i}}{\omega\mu_0}\tan\phi_i{}^{-}E_{x}{}^{i}e^{-jk_1{}^{i}z} + \frac{k_2{}^{i}}{\omega\mu_0}{}^{-}E_{y_2}{}^{i}e^{-jk_2{}^{i}z}\\
{}^{-}H_{y}{}^{i} &= -\frac{k_1{}^{i}}{\omega\mu_0}{}^{-}E_{x_1}{}^{i}e^{-jk_1{}^{i}z} - \frac{k_2{}^{i}}{\omega\mu_0}\tan\phi_i{}^{-}E_{y_2}{}^{i}e^{-jk_2{}^{i}z}.
\end{aligned}
\tag{10}
$$

Superposing (9) and (10), we have for the total component electric and magnetic fields in the ith layer:

$$
\begin{aligned}
H_x{}^{i}{}_{\mathrm{TOTAL}} &= \frac{k_1{}^{i}}{\omega\mu_0}({}^{+}E_{x_1}{}^{i}e^{jk_1{}^{i}z} - {}^{-}E_{x_1}{}^{i}e^{-jk_1{}^{i}z})\tan\phi_i + \frac{k_2{}^{i}}{\omega\mu_0}({}^{-}E_{y_2}{}^{i}e^{-jk_2{}^{i}z} - {}^{+}E_{y_2}{}^{i}e^{jk_2{}^{i}z})\\
H_y{}^{i}{}_{\mathrm{TOTAL}} &= \frac{k_1{}^{i}}{\omega\mu_0}({}^{+}E_{x_1}{}^{i}e^{jk_1{}^{i}z} - {}^{-}E_{x_1}{}^{i}e^{-jk_1{}^{i}z}) - \frac{k_2{}^{i}}{\omega\mu_0}\tan\phi_i({}^{-}E_{y_2}{}^{i}e^{-jk_2{}^{i}z} - {}^{+}E_{y_2}{}^{i}e^{jk_2{}^{i}z})\\
E_x{}^{i}{}_{\mathrm{TOTAL}} &= ({}^{+}E_{x_1}{}^{i}e^{jk_1{}^{i}z} + {}^{-}E_{x_1}{}^{i}e^{-jk_1{}^{i}z}) + \tan\phi_i({}^{+}E_{y_2}{}^{i}e^{jk_2{}^{i}z} + {}^{-}E_{y_2}{}^{i}e^{-jk_2{}^{i}z})\\
E_y{}^{i}{}_{\mathrm{TOTAL}} &= -({}^{+}E_{x_1}{}^{i}e^{jk_1{}^{i}z} + {}^{-}E_{x_1}{}^{i}e^{-jk_1{}^{i}z})\tan\phi_i + ({}^{+}E_{y_2}{}^{i}e^{jk_2{}^{i}z} + {}^{-}E_{y_2}{}^{i}e^{-jk_2{}^{i}z}).
\end{aligned}
\tag{11}
$$

From (11a) through (11d), the boundary conditions may be expressed in matrix form:

$$\left| T^{i+1} \right| \cdot \begin{vmatrix} +E_{x_1}{}^{i+1} \\ -E_{x_1}{}^{i+1} \\ +E_{y_2}{}^{i+1} \\ -E_{y_2}{}^{i+1} \end{vmatrix} = \left| T^{i} \right| \cdot \begin{vmatrix} +E_{x_1}{}^{i} \\ -E_{x_1}{}^{i} \\ +E_{y_2}{}^{i} \\ -E_{y_2}{}^{i} \end{vmatrix} \tag{12}$$

where

$$T^i = \begin{vmatrix} k_1{}^i \tan \phi_i e^{jk_1{}^i z_i} & k_1{}^i \tan \phi_i e^{-jk_1{}^i z_i} & -k_2 e^{+jk_2{}^i z_i} & k_2{}^i e^{-jk_2{}^i z_i} \\ k_1{}^i e^{jk_1{}^i z_i} & -k_1{}^i e^{-jk_1{}^i z_i} & k_2{}^i \tan \phi_i e^{jk_2{}^i z_i} & -k_2{}^i \tan \phi_i e^{-jk^i{}_2 z_i)} \\ e^{jk_1{}^i z_i} & e^{-jk_1{}^i z_i} & \tan \phi_i e^{jk_2{}^i z_i} & \tan \phi_i e^{-jk_2{}^i z_i} \\ -\tan \phi_i e^{jk_1{}^i z_i} & -\tan \phi_i e^{-jk_1{}^i z_i} & e^{jk_2{}^i z_i} & e^{-jk_2{}^i z_i} \end{vmatrix} \tag{13}$$

and where T^{i+1} is identical except that all *superscripts* are changed to $i+1$ and the subscripts on ϕ only are changed to $i+1$.

The column vector for the electric fields in the $i+1$ layer may be determined from (13)

$$\begin{vmatrix} +E_{x_1}{}^{i+1} \\ -E_{x_1}{}^{i+1} \\ +E_{y_2}{}^{i+1} \\ -E_{y_2}{}^{i+1} \end{vmatrix} = \left| T^{i+1} \right|^{-1} \cdot \left| T^{i} \right| \cdot \begin{vmatrix} +E_{x_1}{}^{i} \\ -E_{x_1}{}^{i} \\ +E_{y_2}{}^{i} \\ -E_{y_2}{}^{i} \end{vmatrix}. \tag{14}$$

Matrix equation (14) may now be rewritten, for convenience, as

$$\left| E^{i+1} \right| = \left| A^{i}_{l,m} \right| \cdot \left| E^{i} \right|.$$

The calculation of the $A^i{}_{l,m}$, tensor entries is extremely tedious and will not be presented here.

We now have a matrix which allows us to calculate the fields in the $i+1$ layer from those in the ith, and similarly those in the $i+2$ layer from those in the $i+1$,

$$\left| E^{i+2} \right| = \left| A^{i+1}_{l,m} \right| \cdot \left| A^{i}_{l,m} \right| \cdot \left| E^{i} \right|$$

and so on, for as many layers as comprise the model. This is simply a matrix notation for solving the series of $4 \cdot (n+1)$ equations, corresponding to the boundary conditions at each layer on an n-layered medium, together with the boundary conditions between the surface layer and free space.

For the n-layer model of Figure 2, the origin of the coordinate system has been moved to the interface of the half space. The matrix multiplication giving the fields $|E^n|$ in the nth layer in terms of the $|E^0|$ propagated into the half space has been programmed for an IBM 7094 as has the final matrix operation giving the fields at the surface, $|T^n| \cdot |E^n|$, where $|T^n|$ is the matrix described in (13).

The final matrix, let us denote it by $|B_{l,m}|$, thus relates the field quantities propagated into the half space to those observed at the surface of the model. Since there are no waves traveling in the negative z direction in the basal half space, the ${}^-E_{x_1}{}^0$ and ${}^-E_{y_2}{}^0$ terms in $|E^0|$ are zero and we may then write the following four equations for the field quantities at the surface

$$H_x = B_{11}{}^+E_{x_1}{}^0 + B_{13}{}^+E_{y_2}{}^0 \tag{15a}$$

$$H_y = B_{21}{}^+E_{x_1}{}^0 + B_{23}{}^+E_{y_2}{}^0 \tag{15b}$$

$$E_x = B_{31}{}^+E_{x_1}{}^0 + B_{33}{}^+E_{y_2}{}^0 \tag{15c}$$

$$E_y = B_{41}{}^+E_{x_1}{}^0 + B_{43}{}^+E_{y_2}{}^0. \tag{15d}$$

We may now eliminate ${}^+E_{x_1}{}^0$ and ${}^+E_{y_2}{}^0$ and solve for E_x and E_y in terms of any specified H_x and H_y. In this fashion we may arrive at the impedance entries Z_{ij}, mentioned earlier, relating the E and H fields on the surface on an n-layered anisotropic model. We have used the impedance concept, rather than the admittance, to express the E's as functions of the H's. It may be shown that the incident magnetic polarization is little affected by the geology (anisotropy) and thus the measured H field may be considered as the input to a system of which E is the output.

From equations (15a) (15b) (15c) and (15d) we may also calculate the apparent resistivities in the x and y direction from

$$\rho_{A_x} = \frac{1}{\omega\mu_0}\left|\frac{E_x}{H_y}\right|^2, \quad \text{and}$$

$$\rho_{A_y} = \frac{1}{\omega\mu_0}\left|\frac{E_y}{H_x}\right|^2 \tag{16}$$

for the same model.

$$Z_1 = \left(\frac{\omega\mu_0}{\sigma_1}\right)^{1/2} e^{-i(\pi/4)}, \quad \text{and}$$

$$Z_2 = \left(\frac{\omega\mu_0}{\sigma_2}\right)^{1/2} e^{-i(\pi/4)} \tag{17}$$

and for the single anisotropic layer, sandwiched in isotropic layers, Z_1 and Z_2 are the n-layer impedances that would be measured if the anisotropic layer were to be replaced with an isotropic layer of conductivity σ_1 or σ_2, respectively.

At any other measurement axis angle ϕ (Figure 1) E will be related to H through the following matrix:

$$\begin{vmatrix} E_x \\ E_y \end{vmatrix} = \begin{vmatrix} (Z_1 - Z_2)\sin\phi\cos\phi & Z_2\sin^2\phi + Z_1\cos^2\phi \\ -(Z_2\cos^2\phi + Z_1\sin^2\phi) & -(Z_1 - Z_2)\cos\phi\sin\phi \end{vmatrix} \cdot \begin{vmatrix} H_x \\ H_y \end{vmatrix} \tag{18}$$

DISCUSSION

We will confine the discussion of the impedance tensor to the anisotropic layered models described above, realizing that any model may be characterized in such a fashion. In the case of an anisotropic half space or in the case of an n-layered model possessing only one layer with anisotropic conductivity, the diagonal of the Z_{ij} matrix

$$|E| = |Z_{ij}| \cdot |H|$$

may be reduced to zero by a rotation of the measuring axes. This can be seen in equations (11) where placing ϕ equal to zero leaves two independent sets of equations in the x and y components. Isotropic layers above or below the anisotropic layer have no effect on the zeroing of the diagonal.

Let us consider any model which has been thus reduced. We may then write

$$E_1 = Z_1 H_2$$

$$E_2 = Z_2 H_1$$

where the subscripts refer to measurements made along the σ_1 and σ_2 conductivity axes respectively. For an anisotropic half space the Z's are the characteristic impedances defined by

Several interesting features of anisotropic impedance can be illustrated using this tensor notation alone. For example, if we consider the conditions under which one of the measured electric field components could go to zero, say E_y, we have

$$E_x = Z_{11}H_x + Z_{12}H_y \tag{19}$$

$$0 = Z_{21}H_x + Z_{22}H_y \tag{20}$$

and from (20) this results in

$$\frac{H_x}{H_y} = -\frac{Z_{22}}{Z_{21}}.$$

This is, of course, a trivial result if the matrix happens to be reduced, $Z_{22}=0$, simply indicating that H is linearly polarized perpendicular to E_x; this is also the case if H is oriented along the other conductivity axis and E_x is equal to zero. However, in the general case, when the matrix is not reduced, H_x/H_y need only be a complex quantity equal to $-Z_{22}/Z_{21}$ to obtain a linearly polarized E field. This is a general result not applicable only to anisotropic models.

This situation has a simple implication in the case of an anisotropic half space. The ratio,

$$\frac{Z_{22}}{Z_{21}} = \frac{(Z_1 - Z_2)\sin\phi\cos\phi}{Z_2\cos^2\phi + Z_1\sin^2\phi}$$

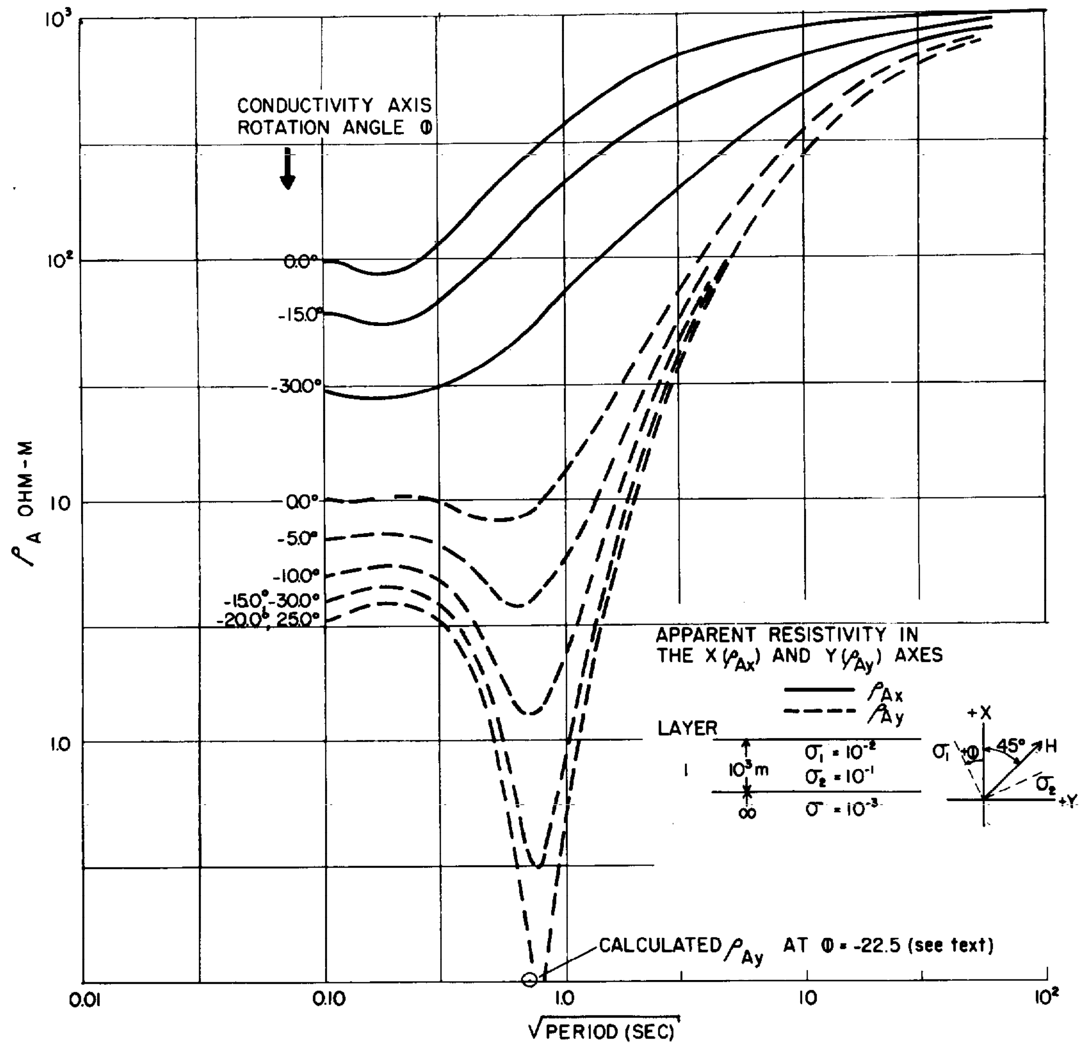

FIG. 3. Apparent resistivities for a two layer anisotropic model.

upon the substitution of relations (17), yields a real number ratio that is independent of frequency. This means that for this model a linearly polarized H can result in a linearly polarized E not at right angles to it. In the multilayered case this ratio will, in general, be complex and will be frequency dependent. However, it is quite possible that for a particular model a linearly polarized H could result in a linearly polarized E not at right angles to it, but only at a particular frequency. It must be noted that this result implies reducing the diagonal to zero only at a particular frequency and not a general relationship such as we have discussed above. If two or more anisotropic layers are present, none of whose conductivity axes are colinear, the diagonal cannot in the general sense be reduced to zero. This has also been pointed out by Mann (1965). The failure of diagonalization attempts to zero the diagonal with field data (Bostick & Smith, 1962) need not imply difficulty with the sources, but rather that effective anisotropy is a function of depth.

This digression into the properties of impedance tensors, which may seem to have been somewhat belabored, allows some interesting observations on a traditionally measured magnetotelluric quantity, the apparent resistivity. Consider the case where $H_x/H_y = -Z_{22}/Z_{21}$, whether in the simple cases discussed or for the general model. Calculations of E_y/H_x will result in values of zero for the apparent resistivity in the y direction. Moreover, for the model in which this ratio depends on frequency and, of course, on ϕ, a very small change in ϕ at the critical frequency could change the ρ_A value by orders of magnitude. This is illustrated in the plot of ρ_A as a function of period and angle ϕ in Figure 3. This particular model did not, in fact, result in a zero for the critical angle of 22.5 degrees and period of approximately 0.7 sec, but rather a minimum. It is quite obvious from this example that large values of the scatter in apparent resistivity results can be accounted for by small changes in orientation of the magnetic field polarization. The rather peculiar arrangement of the linearly polarized H at 45 degrees to the measuring axes, and the conductivity axes "rotating" with respect to the H field is necessitated by the computer program logic which was designed primarily for calculation of the impedance elements.

As a further illustration of these points we have

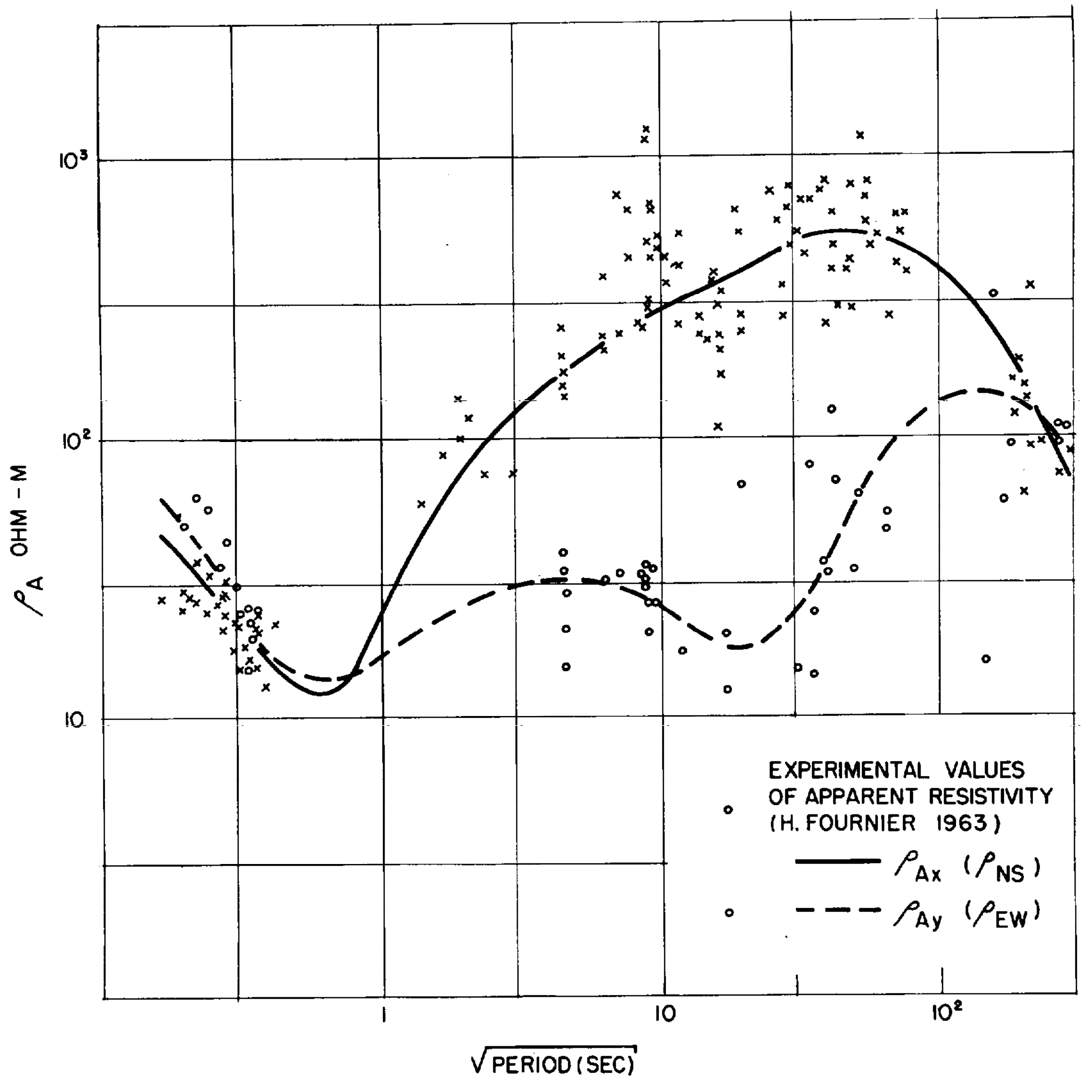

FIG. 4. Experimental values of apparent resistivity [after Fournier (1963)].

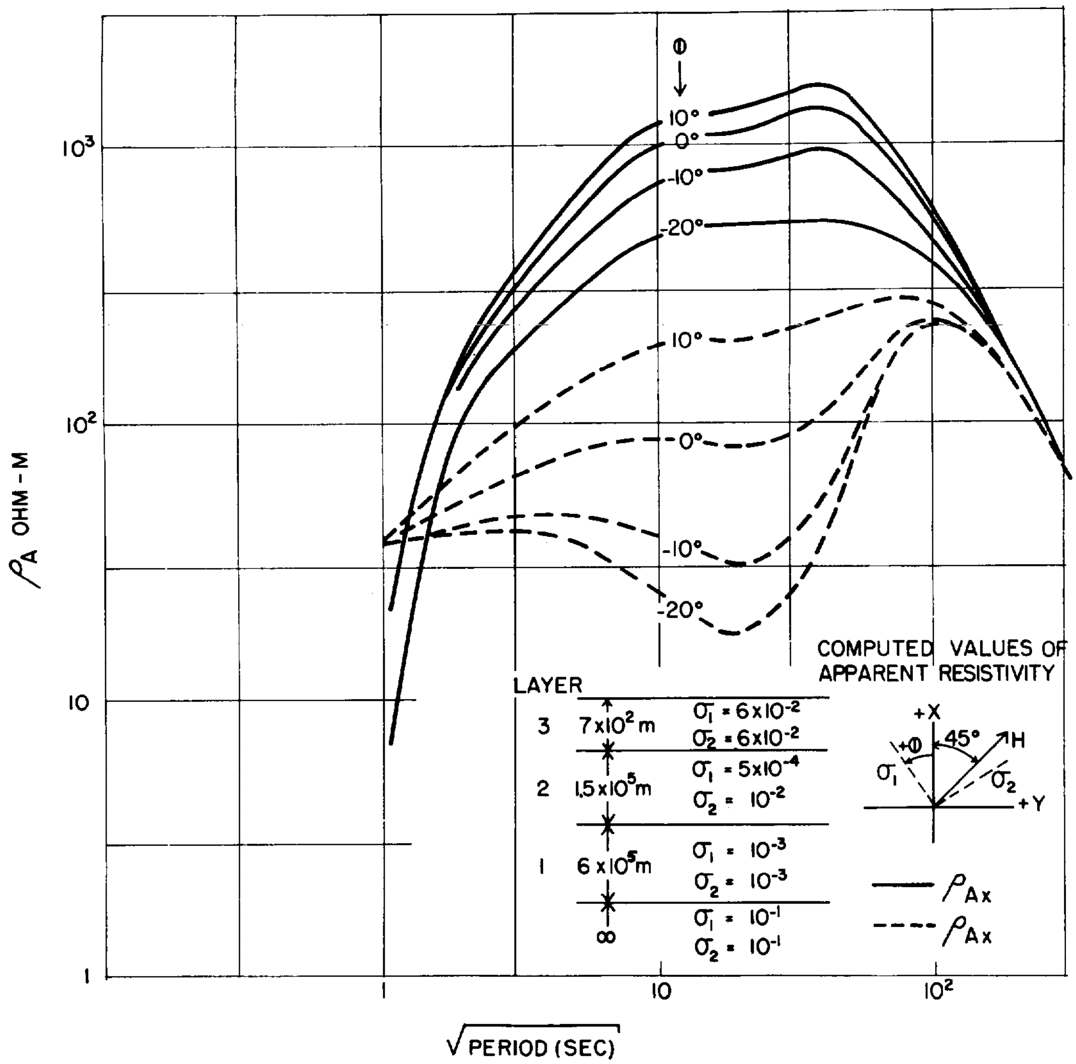

FIG. 5. Apparent resistivity for a four layer anisotropic model.

selected some early data of Fournier (1963b), Figure 4, in which he recognized the existence of a pronounced directional property of the conductivity. Without attempting a detailed interpretation, impossible without the polarization information, we have approximated Fournier's results by the model shown in Figure 5. It is clear that the model results constitute an envelope for the experimental data, but it is also true that such a simple model is at best a rough approximation to the actual geology.

From the above discussion it is evident that measurements of ρ_{A_x} and ρ_{A_y} are not sufficient to characterize the model. In other words, E_y/H_x and E_x/H_y only characterize the ground if it can be shown that the diagonal of the impedance matrix can be reduced to zero, and by computational rotation the E's and H's are thus measured in the principal directions. This also implies, unfortunately, that ρ_A data collected in the past, indicating a conductivity anisotropy, cannot be used in the more detailed interpretation discussed in this paper.

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